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The q-supercoherent states of the q-deformed su(2) superalgebra

Le-Man Kuang

CCAST (World Laboratory), Beijing, People's Republic of China, and (mailing address) Department of Physics, Changsha Normal University of Water Resources and Electric Power, Hunan 410077, People's Republic of China

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Abstract. The q-supercoherent states associated with the q-deformed su(2) superalgebra are constructed explicitly, and their properties are investigated in detail. The completeness relation of the q-supercoherent states is proved by the use of the q-integration defined in this paper.

It is well known that the usual coherent states [1–4] of Lie (super) algebras have wide applications to various branches of physics. Over the past few years, quantum groups and their representations [5, 6] have drawn considerable attention from mathematicians and physicists. A problem of interest is the consideration of coherent states associated with quantum groups, called q-coherent states. Recently q-coherent states of the q-harmonic oscillator [7-9] and the $su_q(2)$ [10] have been investigated independently by several authors. In this paper we propose the q-supercoherent states of the q-deformed su(2) superalgebra [11] which is the simplest of the q-deformed superalgebras [12] and related to integrable models [13].

For the q-deformed su(2) superalgebra, we define for q real

$$[x]=\frac{q^{\frac{1}{2}x}-q^{-\frac{1}{2}x}}{q^{\frac{1}{2}}-q^{-\frac{1}{2}}}\qquad [x]_+=\frac{q^{\frac{1}{2}x}+q^{-\frac{1}{2}x}}{q^{\frac{1}{2}}+q^{-\frac{1}{2}}}\qquad [x]_q=\frac{q^x-q^{-x}}{q-q^{-1}}\tag{1}$$

then

$$[x][x]_+=[x]_q\qquad [2x]=(q^{\frac{1}{2}}+q^{-\frac{1}{2}})[x]_q.\tag{2}$$

We define a q-binomial

$$(a+x)_q^m=\sum_{n=0}^{m}\begin{bmatrix} m\\ n\end{bmatrix}_q a^{m-n}x^n\tag{3}$$

where the q-binomial coefficient is defined by

$$\begin{bmatrix} m\\ n\end{bmatrix}_q=\begin{cases}\dfrac{[m]_q!}{[m-n]_q![n]_q!} & 0\leqslant n\leqslant m\\ 0 & \text{otherwise}\end{cases}\tag{4}$$

with $[m]_q!=[m]_q[m-1]_q[m-2]_q\ldots[1]_q$.

Notice that the q-binomials do not satisfy the product rule of the usual binomials. From the definition of q-binomials one can show

$$(a+q^nx)(a+q^{-1}x)_q^n=(a+x)_q^{n+1}\tag{5}$$

i.e.

$$\frac{(a+q^n x)}{(a+x)_q^{n+1}}=\frac{1}{(a+q^{-1}x)_q^n}. \tag{6}$$

For quantum groups, the q-derivative is defined to be [8, 9]

$$\frac{\mathrm{d}}{\mathrm{d}_q x}f(x)=\frac{f(q^{-1}x)-f(qx)}{q^{-1}x-qx}. \tag{7}$$

If functions $f(x)$ and $F(x)$ satisfy the relation

$$\frac{\mathrm{d}}{\mathrm{d}_q x}F(x)=f(x) \tag{8}$$

then $F(x)$ is called the q-integration of the function $f(x)$, denoted by

$$\int f(x)\,\mathrm{d}_q x=F(x)+C \tag{9}$$

where C is an arbitrary constant. This shows that the operators of q-differentiation and q-integration are inverse to each other.

In order to construct the q-supercoherent states of the q-deformed su(2) superalgebra, we have to establish the following two theorems.

Theorem 1.

$$\frac{\mathrm{d}}{\mathrm{d}_q x}\frac{1}{(a+x)_q^m}=-\frac{[m]_q}{(a+x)_q^{m+1}} \qquad m\in Z^+. \tag{10}$$

It is difficult to verify directly this theorem from the definition of the q-derivative, so we adopt an induction method to prove it.

Proof. When $m=1$, from the definition of the q-derivative (7) one can obtain easily

$$\frac{\mathrm{d}}{\mathrm{d}_q x}\frac{1}{(a+x)_q}=-\frac{1}{a^2+[2]_q x+x^2}=-\frac{1}{(a+x)_q^2} \tag{11}$$

so the theorem holds in this case. Let us assume that for $m=n$, the theorem is correct, i.e.

$$\frac{\mathrm{d}}{\mathrm{d}_q x}\frac{1}{(a+x)_q^n}=-\frac{[n]_q}{(a+x)_q^{n+1}}. \tag{12}$$

We then consider the q-differentiation when $m=n+1$. Taking the q-derivative for both sides of (6), we have

$$\frac{q^n}{(a+qx)_q^{n+1}}+(a+q^{n+1}x)\frac{\mathrm{d}}{\mathrm{d}_q x}\frac{1}{(a+x)_q^{n+1}}=\frac{\mathrm{d}}{\mathrm{d}_q x}\frac{1}{(a+q^{-1}x)_q^n} \tag{13}$$

where we have used the property of the q-derivative

$$\frac{\mathrm{d}}{\mathrm{d}_q x}\left\{\frac{f(x)}{g(x)}\right\}=\{g(q^{-1}x)g(qx)\}^{-1}\left\{g(q^{-1}x)\frac{\mathrm{d}}{\mathrm{d}_q x}f(x)-f(qx)\frac{\mathrm{d}}{\mathrm{d}_q x}g(x)\right\}. \tag{14}$$

Because of the assumption that the theorem holds for $m = n$, the RHS of (13) may be written as

$$\frac{\mathrm{d}}{\mathrm{d}_q x}\frac{1}{(a+q^{-1}x)_q^n} = -\frac{[n]_q q^{-1}}{(a+q^{-1}x)_q^{n+1}}. \tag{15}$$

From (13) and (15) we obtain

$$\frac{\mathrm{d}}{\mathrm{d}_q x}\frac{1}{(a+x)_q^{n+1}} = -\frac{([n]_q q^{-1}+q^n)}{(a+q^{n+1}x)(a+q^{-1}x)_q^{n+1}} = -\frac{[n+1]_q}{(a+x)^{n+2}}$$

where we have used (5) and

$$[m+n]_q = [m]_q q^n + [n]_q q^{-m}. \tag{16}$$

This means that when $m = n+1$, the theorem is correct. The theorem is then proved by the induction principle.

From this theorem and the definition of q-integration, we can get a useful q-integration,

$$\int_0^\infty \frac{1}{(a+x)_q^m}\,\mathrm{d}_q x = \frac{1}{[m-1]_q a^{m-1}}. \tag{17}$$

Theorem 2.

$$\int_0^\infty \frac{x^n}{(a+x)_q^m}\,\mathrm{d}_q x = \frac{[m-n-2]_q![n]_q!}{[m-1]_q!\,a^{m-n-1}} \qquad (0 \leqslant n \leqslant m-2). \tag{18}$$

Proof. Making use of the following q-integration by parts formula for the LHS of (18),

$$\int_a^b f(qx)\left\{\frac{\mathrm{d}}{\mathrm{d}_q x}g(x)\right\}\mathrm{d}_q x = f(x)g(x)\Big|_a^b - \int_a^b \left\{\frac{\mathrm{d}}{\mathrm{d}_q x}f(x)\right\}g(q^{-1}x)\,\mathrm{d}_q x \tag{19}$$

we have

$$\begin{aligned}
\int_0^\infty \frac{x^n}{(a+x)_q^m}\,\mathrm{d}_q x
&= -\frac{x^n}{[m-1]_q(a+x)_q^{m-1}}\Big|_0^\infty + \frac{[n]_q}{[m-1]_q}\int_0^\infty \frac{x^{n-1}}{(a+x)_q^{m-1}}\,\mathrm{d}_q x \\
&= \dots \\
&= -\frac{[n]_q[n-1]_q\dots[2]_q}{[m-1]_q[m-2]_q\dots[m-n]_q}\,\frac{x}{(a+x)_q^{m-1}}\Big|_0^\infty \\
&\quad + \frac{[n]_q[n-1]_q\dots[2]_q}{[m-1]_q[m-2]_q\dots[m-n]_q}\int_0^\infty \frac{1}{(a+x)_q^{m-n}}\,\mathrm{d}_q x.
\end{aligned} \tag{20}$$

Using (17), from (20) one can obtain (18). The proof is complete.

The q-deformed su(2) superalgebra [11] contains the $su_q(2)$ generators J_+, J_- and J_3, and two odd generators V_+ and V_-. These generators satisfy the commutation and anticommutation relations,

$$\begin{aligned} &[J_3, J_\pm] = \pm J_\pm \qquad [J_+, J_-] = [2J_3] \\ &[J_3, V_\pm] = \pm\tfrac{1}{2}V_\pm \qquad [J_\pm, V_\pm] = 0 \\ &[J_\pm, V_\mp] = -(q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{-\frac{1}{2}}[J_3 \mp \tfrac{1}{2}]_+ V_\pm \\ &\{V_+, V_-\} = [J_3 + \tfrac{1}{2}] + [J_3 - \tfrac{1}{2}] \qquad \{V_\pm, V_\pm\} = \pm 2(q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{\frac{1}{2}} J_\pm . \end{aligned} \tag{21}$$

And its j-representation is defined as

$$\begin{aligned} &J_3|j, m\rangle = 2m|j, m\rangle \qquad J_3|j-\tfrac{1}{2}, m\rangle = 2m|j-\tfrac{1}{2}, m\rangle \\ &J_\pm|j, m\rangle = (q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{-\frac{1}{2}}([2j \mp 2m][2j \pm 2m + 2])^{\frac{1}{2}}|j, m+1\rangle \\ &J_\pm|j-\tfrac{1}{2}, m\rangle = (q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{-\frac{1}{2}}([2j-1 \mp 2m][2j+1 \pm 2m])^{\frac{1}{2}}|j-\tfrac{1}{2}, m+1\rangle \\ &V_\pm|j, m\rangle = (q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{\frac{1}{2}}([j \mp m][j \pm m + 1]_+)^{\frac{1}{2}}|j-\tfrac{1}{2}, m+\tfrac{1}{2}\rangle \\ &V_\pm|j-\tfrac{1}{2}, m\rangle = \pm(q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{\frac{1}{2}}([j \pm m + \tfrac{1}{2}][j \mp m + \tfrac{1}{2}]_+)^{\frac{1}{2}}|j, m+\tfrac{1}{2}\rangle \end{aligned} \tag{22}$$

where $m = -j, -j+1, \ldots, j$ for a given j.

In the irrep space, we use positive-definite metric, then $\langle j, m|j, n\rangle = \delta_{m,n}$, $\langle j-\frac{1}{2}, m|j-\frac{1}{2}, n\rangle = \delta_{m,n}$ and $\langle j, m|j-\frac{1}{2}, n\rangle = 0$. And the resolution of identity is written as

$$\sum_{n=0}^{2j} |j, -j+n\rangle\langle j, -j+n| + \sum_{n=0}^{2j-1} |j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}\rangle\langle j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}| = 1 \tag{23}$$

where 1 is the identity operator.

From the j-representation, we can obtain two useful formula for computation,

$$J_+^n|j, -j\rangle = (q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{\frac{1}{2}n} \begin{bmatrix} 2j \\ n \end{bmatrix}_q^{\frac{1}{2}} [n]_q!|j, -j+n\rangle \tag{24a}$$

$$J_+^n|j-\tfrac{1}{2}, -j+\tfrac{1}{2}\rangle = (q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{\frac{1}{2}n} \begin{bmatrix} 2j-1 \\ n \end{bmatrix}_q^{\frac{1}{2}} [n]_q!|j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}\rangle. \tag{24b}$$

We introduce a q-exponential operator defined by

$$e_q^{zJ_+} = \sum_{n=0}^{\infty} \frac{z^n J_+^n}{[n]_q!}. \tag{25}$$

The q-supercoherent states of the q-deformed su(2) superalgebra are defined as follows:

$$|z, \theta\rangle_q = e_q^{zJ_+} e^{\theta V_+}|j, -j\rangle \tag{26}$$

where $|j, -j\rangle$ is the lowest weight state of the j-representation of the q-deformed superalgebra. Since z is a complex variable and θ a Grassmann variable, $|z, \theta\rangle_q$ are called q-supercoherent states.

Substituting (25) and the expansion of $e^{\theta V_+}$ into (26), we have

$$\begin{aligned} |z, \theta\rangle_q = &\sum_{n=0}^{2j} \begin{bmatrix} 2j \\ n \end{bmatrix}_q^{\frac{1}{2}} (q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{\frac{1}{2}n} z^n |j, -j+n\rangle \\ &+ (q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{\frac{1}{2}}[2j]^{\frac{1}{2}}\theta \sum_{n=0}^{2j-1} \begin{bmatrix} 2j-1 \\ n \end{bmatrix}_q^{\frac{1}{2}} (q^{\frac{1}{2}} + q^{-\frac{1}{2}})^{\frac{1}{2}n} z^n |j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}\rangle. \end{aligned} \tag{27}$$

From this one may obtain the transformation coefficients,

$$\langle j, m|z, \theta\rangle_q = (q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}(j+m)}\begin{bmatrix} 2j \\ j+m \end{bmatrix}_q^{\frac{1}{2}} z^{j+m} \tag{28}$$

$$\langle j-\tfrac{1}{2}, m|z, \theta\rangle_q = [2j](q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}(j+m+\frac{1}{2})}\begin{bmatrix} 2j-1 \\ j+m-\frac{1}{2} \end{bmatrix}_q^{\frac{1}{2}} z^{j+m-\frac{1}{2}}\theta. \tag{29}$$

If we introduce

$$|z\rangle_q^1 = \sum_{n=0}^{2j}\begin{bmatrix} 2j \\ n \end{bmatrix}_q^{\frac{1}{2}}(q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}n}z^n|j, -j+n\rangle \tag{30}$$

$$|z\rangle_q^2 = \sum_{n=0}^{2j-1}\begin{bmatrix} 2j-1 \\ n \end{bmatrix}_q^{\frac{1}{2}}(q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}n}z^n|j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}\rangle \tag{31}$$

then one can express the q-supercoherent states as a concise form,

$$|z, \theta\rangle_q = |z\rangle_q^1 + (q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}}[2j]^{\frac{1}{2}}\theta|z\rangle_q^2. \tag{32}$$

From (30) and (31), one can get the orthogonality relations,

$${}_q^1\langle z'|z\rangle_q^2 = 0 \tag{33}$$

$${}_q^1\langle z'|z\rangle_q^1 = (1+(q^{\frac{1}{2}}+q^{-\frac{1}{2}})z\bar{z}')_q^{2j} \tag{34}$$

$${}_q^2\langle z'|z\rangle_q^2 = (1+(q^{\frac{1}{2}}+q^{-\frac{1}{2}})z\bar{z}')_q^{2j-1}. \tag{35}$$

For two arbitrary q-supercoherent states, $|z, \theta\rangle_q$ and $|z', \theta'\rangle_q$ not orthogonal to each other

$${}_q\langle z', \theta'|z, \theta\rangle_q = (1+(q^{\frac{1}{2}}+q^{-\frac{1}{2}})z\bar{z}')_q^{2j} + (q^{\frac{1}{2}}+q^{-\frac{1}{2}})[2j](1+(q^{\frac{1}{2}}+q^{-\frac{1}{2}})z\bar{z}')_q^{2j-1}\bar{\theta}'\theta \tag{36}$$

so that the q-supercoherent states of the deformed su(2) superalgebra are linearly dependent.

The core of the system of the coherent states is their completeness relation. We now consider the completeness relation of the q-supercoherent states. The problem here consists in finding a weight q-superfunction $\sigma_q(z, \theta)$ such that

$$\int d_q^2 z\, d^2\theta\, \sigma_q(z, \theta)|z, \theta\rangle_{q\,q}\langle z, \theta|$$
$$= \sum_{n=0}^{2j}|j, -j+n\rangle\langle j, -j+n| + \sum_{n=0}^{2j-1}|j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}\rangle\langle j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}|$$
$$= 1 \tag{37}$$

with

$$d_q^2 z = r\, d_q r\, d\varphi \qquad z = r\, e^{i\varphi} \qquad d^2\theta = d\bar{\theta}\, d\theta. \tag{38}$$

Note that the integral over $d_q r$ is a q-integration while the integration over φ is the usual integration.

Although the weight q-superfunction $\sigma_q(z, \theta)$ is expanded generally on θ as the sum of the four terms, it can be shown that only two terms have contribution to the q-integration on the LHS of (37),

$$\sigma_q(z, \theta) = A_q(z) + B_q(z)\bar{\theta}\theta. \tag{39}$$

Substituting (39) into the LHS of (37) and integrating over the Grassmann variable, we have

$$\iint d_q^2 z\, d^2\theta\, \sigma_q(z,\theta)|z,\theta\rangle_{q\,q}\langle z,\theta|$$

$$= \int d_q^2 z\{B_q(z)|z\rangle_q^1\,{}_q^1\langle z| + (q^{\frac{1}{2}}+q^{-\frac{1}{2}})[2j]A_q(z)|z\rangle_q^2\,{}_q^2\langle z|\} \tag{40}$$

where we have used Grassmann integrals

$$\int d\theta = \int d\bar{\theta} = 0 \qquad \int \theta\, d\theta = \int \bar{\theta}\, d\bar{\theta} = 1. \tag{41}$$

We shall now determine $A_q(z)$ and $B_q(z)$. Let $|f\rangle$ and $|g\rangle$ be two arbitrary vectors. Then (37) means that

$$\langle f|g\rangle = \langle f| \int d_q^2 z\{B_q(z)|z\rangle_q^1\,{}_q^1\langle z| + (q^{\frac{1}{2}}+q^{-\frac{1}{2}})[2j]A_q(z)|z\rangle_q^2\,{}_q^2\langle z|\}|g\rangle. \tag{42}$$

Substituting (30), (31) and (38) into (42), we have

$$\begin{aligned}
\langle f|g\rangle = \langle f| \int_0^\infty d_q r \int_0^{2\pi} d\varphi \Bigg\{ &\sum_{n,m=0}^{2j} \begin{bmatrix} 2j \\ n \end{bmatrix}_q^{\frac{1}{2}} \begin{bmatrix} 2j \\ m \end{bmatrix}_q^{\frac{1}{2}} (q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}(n+m)} B_q(r) r^{n+m+1} \\
&\times e^{i(n-m)\varphi}|j,-j+n\rangle\langle j,-j+m| + (q^{\frac{1}{2}}+q^{-\frac{1}{2}})[2j] \sum_{n,m=0}^{2j-1} \begin{bmatrix} 2j-1 \\ n \end{bmatrix}_q^{\frac{1}{2}} \begin{bmatrix} 2j-1 \\ m \end{bmatrix}_q^{\frac{1}{2}} \\
&\times (q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}(n+m)} A_q(r) r^{n+m+1}\, e^{i(n-m)\varphi}|j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}\rangle \\
&\times \langle j-\tfrac{1}{2}, -j+m+\tfrac{1}{2}| \Bigg\}|g\rangle \\
= \langle f| \Bigg\{ 2\pi \sum_{n=0}^{2j} &\int_0^\infty d_q r\, r^{2n+1} \begin{bmatrix} 2j \\ n \end{bmatrix}_q (q^{\frac{1}{2}}+q^{-\frac{1}{2}})^n B_q(r)|j,-j+n\rangle\langle j,-j+n| \\
&+ \sum_{n=0}^{2j-1} (q^{\frac{1}{2}}+q^{-\frac{1}{2}})[2j]2\pi \int_0^\infty d_q r\, r^{2n+1} \begin{bmatrix} 2j-1 \\ n \end{bmatrix}_q \\
&\times (q^{\frac{1}{2}}+q^{-\frac{1}{2}})^n A_q(r)|j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}\rangle\langle j-\tfrac{1}{2}, -j+n+\tfrac{1}{2}| \Bigg\}|g\rangle.
\end{aligned} \tag{43}$$

Hence we must have

$$2\pi \begin{bmatrix} 2j \\ n \end{bmatrix}_q \int_0^\infty d_q r\, r^{2n+1}(q^{\frac{1}{2}}+q^{-\frac{1}{2}})^n B_q(r) = 1$$

$$2\pi[2j](q^{\frac{1}{2}}+q^{-\frac{1}{2}}) \begin{bmatrix} 2j-1 \\ n \end{bmatrix}_q \int_0^\infty d_q r\, r^{2n+1}(q^{\frac{1}{2}}+q^{-\frac{1}{2}})^n A_q(r) = 1.$$

Namely,

$$\int_0^\infty d_q r\, r^{2n+1}(q^{\frac{1}{2}}+q^{-\frac{1}{2}})^n B_q(r) = \frac{[2j-n]_q![n]_q!}{2\pi[2j]_q!} \tag{44}$$

$$\int_0^\infty d_q r\, r^{2n+1}(q^{\frac{1}{2}}+q^{-\frac{1}{2}})^n A_q(r) = \frac{[2j-n-1]_q![n]_q!}{2\pi[2j](q^{\frac{1}{2}}+q^{-\frac{1}{2}})[2j-1]_q!}. \tag{45}$$

Using theorem 2, from (44) and (45) we obtain

$$B_q(r)=\frac{[2]_q[2j+1]_q(q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}}}{2\pi(1+(q^{\frac{1}{2}}+q^{-\frac{1}{2}})r^2)_q^{2j+2}} \tag{46a}$$

$$A_q(r)=\frac{[2]_q[2j]_q}{2\pi[2j](1+(q^{\frac{1}{2}}+q^{-\frac{1}{2}})r^2)_q^{2j+1}}. \tag{46b}$$

Therefore the weight q-superfunction $\sigma_q(z,\theta)$ is given by

$$\sigma_q(z,\theta)=\frac{[2]_q}{2\pi}\left\{\frac{[2j]_q}{[2j](1+(q^{\frac{1}{2}}+q^{-\frac{1}{2}})z\bar{z})_q^{2j+1}}+\frac{[2j+1]_q(q^{\frac{1}{2}}+q^{-\frac{1}{2}})^{\frac{1}{2}}}{(1+(q^{\frac{1}{2}}+q^{-\frac{1}{2}})z\bar{z})_q^{2j+2}}\bar{\theta}\theta\right\}. \tag{47}$$

With the aid of the completeness relation of the q-supercoherent states, one can expand an arbitrary vector $|\psi\rangle$ as

$$|\psi\rangle=\iint \mathrm{d}_q^2 z\,\mathrm{d}^2\theta\,\sigma_q(z,\theta)|z,\theta\rangle_{q\,q}\langle z,\theta|\psi\rangle. \tag{48}$$

Setting $|\psi\rangle=|z',\theta'\rangle_q$, an arbitrary q-supercoherent state of the q-deformed su(2) superalgebra, then we have

$$|z',\theta'\rangle_q=\iint \mathrm{d}_q^2 z\,\mathrm{d}^2\theta\,\sigma_q(z,\theta)|z,\theta\rangle_{q\,q}\langle z,\theta|z',\theta'\rangle_q. \tag{49}$$

This means that the system of q-supercoherent states is actually overcomplete since it contains subsystems of q-supercoherent states which are complete systems.

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